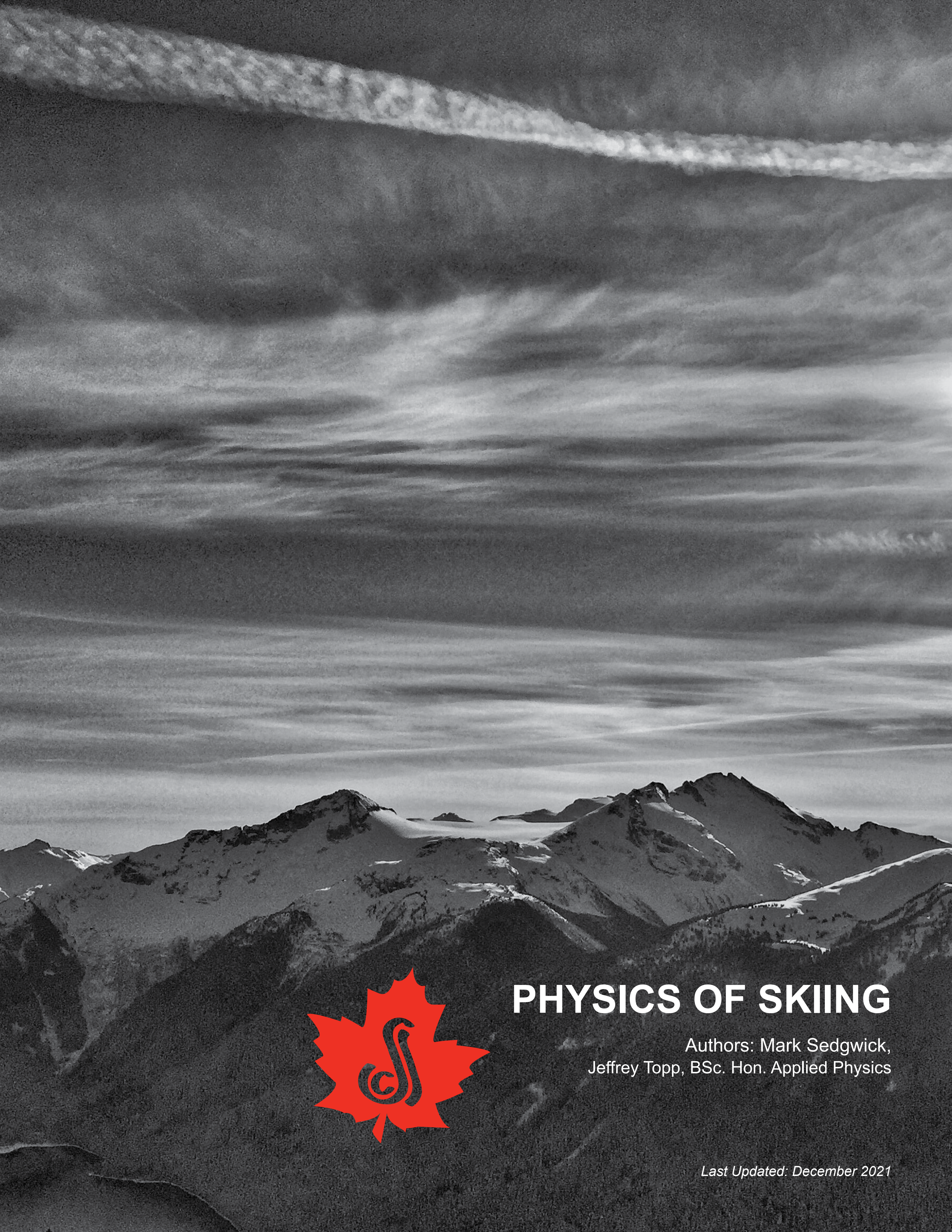


PHYSICS OF SKIING

Authors: Mark Sedgwick,
Jeffrey Topp, BSc. Hon. Applied Physics

Last Updated: December 2021

Table of Contents

Introduction

The Physics of Skiing is the foundational document of the CSIA Technical Concepts. The Skills Framework and Performance Model follow. The Physics of Skiing explains the laws of motion and how they impact us when we ski — the ground rules. The Skills Framework builds upon these ground rules and explores what our skis can do and how our bodies move to do it — the components affecting our motion. Finally, the Performance Model discusses skiing concepts necessary to achieve specifically desired outcomes for the first-time skier through to the expert skier and puts it all together.

The alpine ski environment provides endless variations in terrain and conditions. Skiers use their equipment to manage their speed and direction in this ever-changing landscape. An understanding of the physics behind skiing clarifies the rules of what must happen when our skis interact with the snow and ultimately what then happens when the snow pushes back on us with a reaction force, which we refer to as the ground reaction force or GRF.

The ground rules are based on Newtonian mechanics (also called classical mechanics). With a knowledge of motion and the forces that control it, we realize skiing is deterministic. Specific forces will produce determined outcomes. Therefore, an instructor is able to both develop appropriate techniques and make decisions that extract the required forces from the snow for individual students in a wide variety of situations. This knowledge also ensures the language we use across our organization is both accurate and consistent.

In this document, forces are analyzed from a snow to ski interface perspective consistent with the Canadian Ski Instructors Alliance's emphasis on *function over form.*

The knowledge and fundamental understanding of how skiing works presented in this technical document aren't necessarily what we teach our learners, it is for your knowledge as a professional. It will equip you with the intellectual concepts of how skiing works and the confidence to teach. This underlying knowledge will help guide and influence the design of *what* you teach. *How* you teach, or the delivery of what you teach, lies in the CSIA teaching concept of the Collaborative Teaching Approach (CTA).

Once you understand the role of force in skiing, everything else will make much more sense.

A List of Definitions

Acceleration – the rate of change of velocity with respect to time (units = m/s^2).

Force – strength or energy exerted (a push or a pull) on an object (units = N (Newtons).

Inertia – a property of matter by which it continues in its existing state of rest or uniform motion in a straight line, unless that state is changed by an external force.

Impulse – a force acting briefly on a body and producing a finite change of momentum. Expressed mathematically, the product of force × time (units = Ns).

Momentum – the quantity of motion of a moving body. Expressed mathematically, product of mass × velocity (units = Kg m/s).

Net force – the sum of all forces that act upon an object. The net force is the single force that represents the effect of all forces on an object.

Pressure – a measurement of force per unit area (units = the pascal (Pa), which is equivalent to N/m^2).

Speed – the rate at which an object moves (units = m/s).

Vector – a quantity having direction as well as magnitude, commonly represented by a directed line segment whose length represents magnitude and whose orientation in space represents the direction (units = units of the quantity it is describing).

Velocity – the speed of an object together with a directional component (units = m/s together with a direction).

Weight – the force with which a body is attracted towards earth (units = kg).

Speed versus velocity – speed is the rate at which an object is moving, while velocity is the rate and direction of an object's movement.

The brain symbol indicates further technical analysis on the topic at hand. When you see this, look in the Additional Information section on page 25 for further knowledge.

CHAPTER 1: Force – What It Is and Why It's Important to Skiing

1.1 Forces

Skiing is motion and motion is caused by force. A stationary object does not move unless a force **(a push or pull)** acts on it to put it in motion. Once it is moving, it carries on at the same speed and in the same direction unless another force makes it speed up, change direction, slow down or stop.

While skiing, we are pulled down the hill by the force of gravity, due to the low friction interface between the snow and our ski bases. Without the introduction of any additional forces, we would simply go straight down the hill. Turning and slowing forces are necessary to control our speed and direction.

Whenever there is an interaction between two objects – in this case, the ski and the snow – there is a force that acts upon each of the objects. These forces make the ski turn or not turn, speed up, slow down or maintain the same speed. They are also responsible for the thrill or sense of impending doom we feel on the hill. Some forces, such as gravity, we cannot control, but other forces, we can control. With a reasonable knowledge of these forces and the movements required to manage them, we can influence the outcome of the ski/snow interaction and our path down the hill.

Takeaways:

1. Skiing is motion.
2. We cannot have motion without force.
3. A force is a push or pull upon an object resulting from the object's interaction with another object (ski and snow).
4. The force of gravity pulls us down the hill.
5. Other forces facilitate speed and direction control.

1.2 Vectors

Let's delve a little into the personality traits of a force. Forces are vectors, which simply means they have a two-faceted nature, namely **direction and magnitude.** It's important to understand this characteristic of a force in that the concept of a vector often comes into play in skiing. Geometrically, we can picture a vector as a directed line segment whose length is the magnitude of the vector with an arrow indicating its direction. Force is one example of a vector, discussed in this section. Another example of a vector is velocity, which also has both magnitude and direction. Velocity is often incorrectly used interchangeably with speed, which simply has magnitude, but no direction.

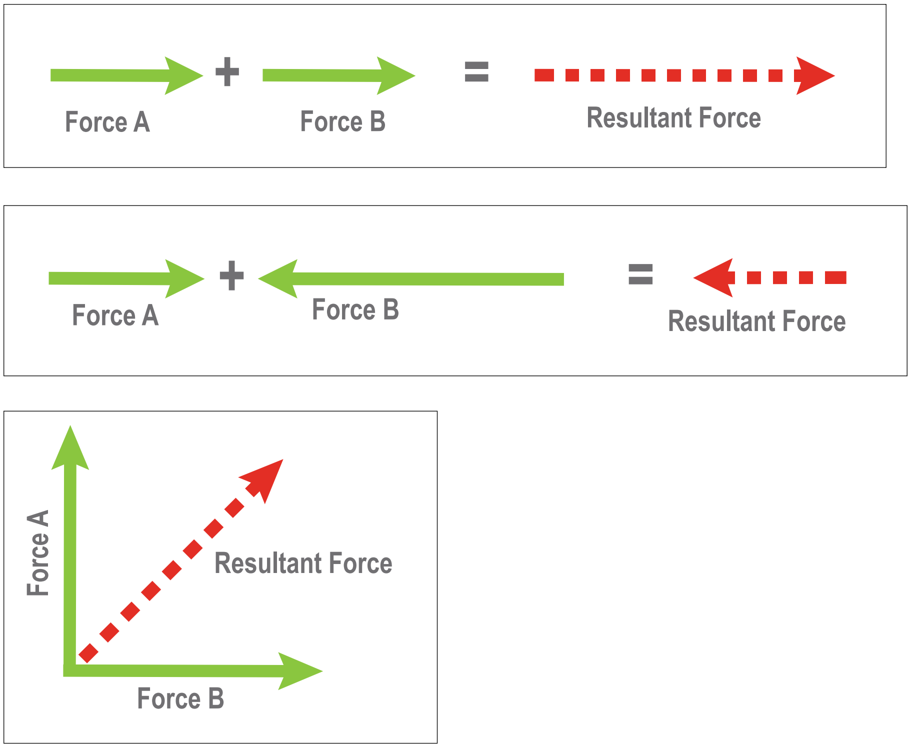
In physics, acceleration has a speed component but also a directional component. Therefore, it is also a vector. An object is accelerating if it is changing direction, even if remains at a constant speed, such as an object travelling in a circle at a constant speed accelerating towards the centre of the circle. If a body is changing direction, it is accelerating.

Example 1: A skier who is carving a turn at constant speed is accelerating – his or her direction is changing!

Example 2: A carved turn is an example of when direction changes, but not necessarily magnitude (speed).

Forces are vectors and vectors are additive; thus, forces are additive. Forces are added like lines, taking length and angle (direction) into account. Adding forces is the same as combining them. When two forces act in the same direction, they are added together. When forces act in opposite directions, they are combined by subtracting the smaller force from the larger force. When the magnitude and direction of all forces are combined, they yield a resultant force. We use a free body diagram to show the relative magnitude and direction of all forces acting upon an object in a given situation.

Figure 1. Free body diagrams



Takeaways:

1. Forces have two key properties, magnitude and direction.
2. In physics, acceleration has a speed component, but also a directional component.
3. An object is accelerating if it is changing direction, even if remains at a constant speed.
4. If a body is changing direction, it is accelerating.
5. Forces are additive.

1.3 Internal and External Forces

Forces in skiing can be divided into two categories: internal and external. Internal forces are generated by the skier using their muscles. We can utilize these forces to influence movement of our body and ultimately the ski.

Example: Internal and external leg rotation and flexion and extension are examples of internal forces. Let rotation assists with the rotational control actions of the ski and flexion/extension assists with the pressure control actions of the ski.

External forces are acting on the skier from outside the body. These forces affect the skier and are responsible for moving the skier.

Example: The force of gravity and friction between the snow and the skis are examples of external forces.

Takeaways:

1. Forces in skiing can be divided into two categories: internal and external.
2. Internal forces are generated by the skier using their muscles. We can utilize these forces to influence our movements.
3. External forces are acting on the skier from outside the body. These forces are responsible for moving the skier.

CHAPTER 2: Motion (Newtonian Mechanics)

2.1 Newton's Laws of Motion – The Ground Rules

The rules of skiing are not determined by individual ski organizations, they are governed by the universal laws of motion, as is every object interacting with any other object on our planet. Sir Isaac Newton elegantly defined the laws of motion well over 300 years ago. These laws determine the ground rules of skiing – there is no debate on this.

Newton's three laws of motion lay the foundation of classical mechanics. They describe the relationship between a body and the forces acting upon it, and its motion in response to those forces. The laws governing how the Earth orbits the sun are the same laws governing our motion on skis.

The Ground Rules:

Newton's first law states that a body at rest will remain at rest and a body in motion will remain in motion, unless acted upon by an external, unbalanced force. This law is Sir Isaac Newton's attempt to define what we commonly refer to as inertia. Inertia can also be described as a force which keeps stationary objects at rest and moving objects in motion at constant velocity.

If we ski at a constant speed in a straight line, this condition will remain unless other forces are introduced. The various forces acting upon us, such as gravity (a constant on earth), as well as those we generate (and can manipulate using our muscles and equipment), are very relevant in terms of impacting our motion.

Newton's second law, sometimes referred to as the law of force and acceleration, states that force is equal to the product of mass times acceleration (F=ma). The forces discussed in Newton's first law are governed by this formula. If a body has a net force acting upon it, it is accelerated in accordance with the equation. Conversely, if a body is not accelerated, there is no net force acting upon it. Simply said, how much something moves is proportional to the size of the force acting on it.

Newton's second law allows us to govern the forces acting upon us. By manipulating our skis on the snow, we can manage these forces and therefore control our motion to achieve our objectives.

Example: A world cup ski racer and a recreational skier both strive to control their acceleration (speed and direction), however, the intensity of the forces that each must manage are very different in light of their objectives. The objective of the racer is to generate speed while running a course, whereas the recreational skier seeks to control speed. Both look to control direction. As speed increases or decreases, so do the forces that must be managed. This concept is discussed in more detail in later sections.

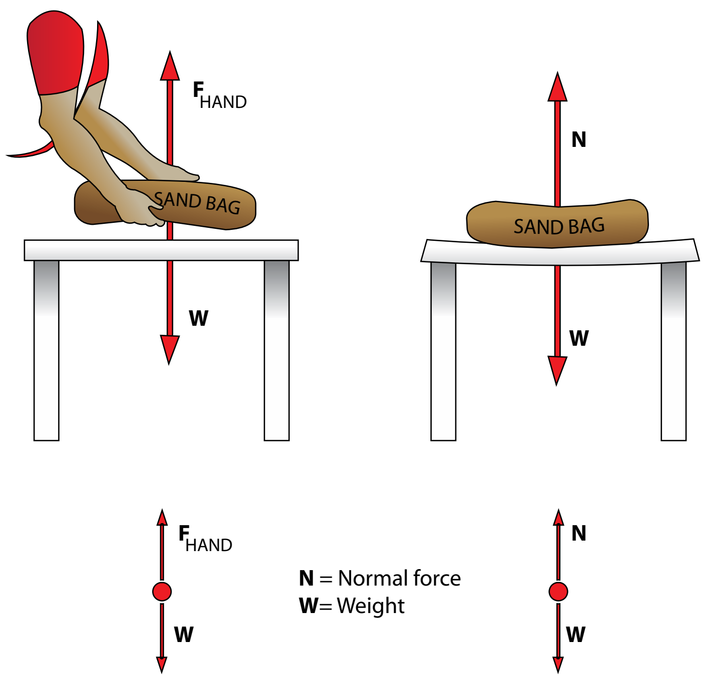
Additional information (1)

Finally, Newton's third law states that for every action, there is an equal, but opposite, reaction. Put another way, when you push on something, it pushes back just as hard.

FA = - FB. When skiers use their muscles or equipment to push on the snow, the snow pushes back!

It makes sense to most people that a person would have to exert an upward force with their hands when carrying a heavy bag of sand, as seen in diagram (a) of Figure 2 below.

Figure 2. Normal force diagram



Now, if we place that sandbag on a table, does the table exert the same force we used to hold up the sandbag? Can a table really exert an upward force on the bag of sand and, by extension, can snow really exert a force on us? Perhaps the table in diagram (b) above is simply stopping the bag of sand from falling down?

However, that is not the case. If there was only a downward force of gravity on the sandbag, the sandbag would accelerate downward. Recall Newton's second law: F=ma, or rearranged, a=F/m.

For example, acceleration is proportional to force applied. Therefore, the table must exert an upward force to prevent the sandbag from falling to the floor. We refer to this upward force as the normal force (N), as depicted above.

In physics, the normal force is the force that a surface exerts to prevent solid objects from passing through each other, this force is always perpendicular to the contacting surfaces.

Takeaways:

1. The laws of motion govern the laws of skiing. Newton's laws define the rules of motion.
2. Newton's first law helps us define inertia, a force which keeps stationary objects at rest and moving objects in motion at constant velocity.
3. Newton's second law tells us that how much something moves is proportional to the size of the force acting on it. By manipulating our skis on the snow, we can manage the forces acting on us and therefore control our motion in order to achieve our objectives.
4. When skiers use their muscles or equipment to push on the snow, Newton's third law tells us the snow pushes back. So, yes, the snow really does push on us!

CHAPTER 3: Forces and Motion

In this chapter, we shall familiarize ourselves with the key forces and concepts related to skiing.

Gravity holds us to the planet, but is also the force the propels us on skis. While in motion, we have momentum. We can manipulate this momentum to our advantage (turning and speed control) through our skis.

3.1 The Centre of Mass (COM)

The centre of mass is a specific point in the body where we consider the mass of the skier and equipment to be concentrated. It is generally located just below the navel, but moves as we change our body position.

We use the COM concept to represent a single point of an entire system (skier and equipment in our case) when considering the effects of force, motion, and balance. All forces act on the centre of mass (COM) when considering the trajectory of a body as a whole.

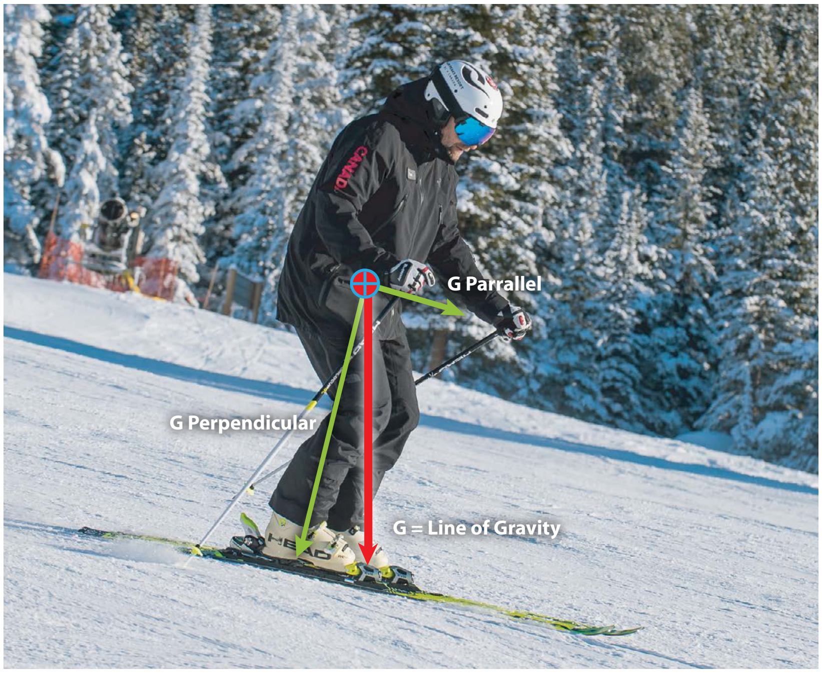
Figure 3. COM

3.2 Gravity

On a slope, gravity has two components: a component that pulls on us in a direction perpendicular to the surface of the snow (the part that holds us to the planet), and a component that pulls us down the hill. The relative strength of these two components varies with the incline of the slope. On a steep slope, there is more pull down the slope and less towards the snow. On a flatter slope, there is less pull down the hill and a stronger pull towards the surface of the snow.

In the figure below. G represents the gravity line. This line points to the Earth's centre. G-perpendicular represents the component of gravity perpendicular to the surface of the snow and G-parallel represents the component of gravity parallel to the slope.

Figure 4.



3.3 Friction

Friction always opposes motion and serves to slows us down. The two relevant occurrences in skiing are the friction between the ski base and the snow and the air friction (air resistance) that pushes against a skier's body. The friction of the snow changes quickly with snow texture, depth and terrain contours. Snow friction is greater in drifted turns than a clean, carved turn. Well-tuned skis also reduce snow friction. The effect of air friction is affected by the wind - increasing with a headwind and decreasing with a tailwind. Exposing less head-on surface area for the wind to drag on reduces air friction, like moving into a tuck position, for example.

Figure 5. Friction

Takeaways:

1. The centre of mass is a specific point in the body where we consider the mass of the skier and equipment to be concentrated.
2. Gravity is the primary force that provides skiers with momentum.
3. Gravity acts as two components: a component that pulls on us in a direction perpendicular to the surface of the snow and a component that pulls the COM down the hill.
4. Friction slows us down. We experience two types of friction: the friction between the ski and the snow and the air friction that pushes against a skier's body.

3.4 Momentum

Momentum is an especially valuable concept for us when it comes to skiing. We can use the concept of momentum to predict and understand the outcome of interactions between the snow and our skis.

Momentum defines how much motion an object has, defined by Newton as the "quantity of motion". It defines the **mass and the velocity** of an object (a mass that is in motion). Momentum can be expressed as the product of mass x velocity. An object that is on the move has momentum, the more an object is on the move the more momentum it has!

Force and momentum are close friends. They are both defined by magnitude and direction. If we are in motion while skiing, we have momentum, and as Newton's first law states, to impact our motion, an external force will be required. The external force in this case is the snow pushing back on our skis. As humans, we feel changes in speed and direction (i.e., a change in velocity), so we feel changes in our momentum.

Accordingly, through changes in velocity which we initiate through the interaction of our skis with the snow, (using rotational, edge and pressure actions), we can manipulate our momentum and therefore the forces acting upon us, i.e., how the snow pushes on us!

You may be asking yourself, what is this important force we extract from the snow? We jump right into that in the following section.

Impulse, another term we often hear and often use incorrectly, is intrinsically tied to the above force and momentum discussion. Impulse is merely a change in momentum. Or put another way, a force applied over a specific time period.

Additional information (2)

Takeaways:

1. Momentum defines how much motion an object has (the quantity of motion).
2. Two variables determine an object's momentum, i.e., mass and velocity.
3. The momentum of an object will only increase or decrease when a net force is acting upon it.
4. On skis, we manipulate our momentum through the interaction of our skis with the snow (using rotational, edge and pressure control actions) and therefore control how the snow pushes on us!

3.5 Ground Reaction Force (GRF)

You may recall from Newton's third law that for every action, there is an equal, but opposite, reaction. As such, **when we push on the snow, the snow pushes back with a reaction force,** much as the table pushed back with the normal force N in Figure 2 above. We refer to this force pushing back on us from the snow as the ground reaction force, or GRF. GRF is the force that provides the sensation of weight or pressure between your skis and the snow and is constantly changing due to terrain variations, ski to snow interaction and our location within a turn.

GRF is the external force of the snow pushing back on our skis as mentioned above and it can be simplified to represent the exact trajectory and magnitude of the snow's push on us. It's the external force required to manage our speed and direction. If you are thinking to yourself, this is important – you're damn right. It's the most valuable concept in this body of work and hence will be the nucleus from which we focus on for the remainder of the document.

Controlling speed, direction and staying upright would be considered by most (with the addition of fun) to encompass skiing's primary objectives. This being the case, the external force we refer to as GRF is of indispensable importance both for your fundamental understanding of skiing and as a mechanism to deduce outcomes from technique. From the perspective of a ski professional, this should be a liberating epiphany.

Skiing competency lies in the ability to manage GRF. Through manipulating the skis' interaction with the snow, we get the snow to push on us in just the right way to achieve our desired outcome. GRF is the force that provides direction and speed control; it allows us to go where we want, at the speed we desire. It is the force that makes turning possible.

Just as the path of a hockey puck is controlled by applying force to it with a stick, a skier's path is controlled by the GRF acting upon our skis.

So yes, the snow pushes on us. In its simplest form, skiing is merely controlling the magnitude and direction of the force of the snow pushing on us, and how we position our bodies to balance against that push. It's Newton's third law, the law of reaction!

As depicted in Figure 6, the snow pushing on the skis makes the skier turn.

Figure 6. The force from the snow pushing against the bottom of the ski makes the ski turn.

Takeaways:

1. Yes, the snow really does push on us.
2. The ground reaction force (GRF) is the snow's reaction to the force we apply to the snow.
3. The GRF can be simplified to represent the exact trajectory of the snow's push on us.
4. The GRF is the force that provides direction and speed control. It is the force that makes turning possible.
5. Through manipulating the GRF we get the snow to push on us in just the right way to achieve our desired outcome.
6. We control GRF through the interaction of our skis with the snow.
7. Gravity is the primary force that moves us. Motion yields momentum and we manage our momentum (speed and direction) through manipulation of GRF.

3.6 GRF: A Balance Perspective

The interface between our ski bases and the snow is very slippery. This low friction interaction is what facilitates the sport of skiing, but it also creates a situation in which we are unaccustomed to in everyday life, that of having much reduced friction at our base of support. There are few situations in life where we must balance atop a moving object; skiing is one such experience that provides us with this sensation. Fortunately, human balance, a complex system of sensors and processing functions, is well suited to navigating such a challenging activity.

Simply stated, balance is the outcome of a favorable base of support and centre of mass relationship. When the forces acting on our center of mass pass through our base of support, we feel stable. Dynamic balance is achieved when all forces on our centre of mass are in a state of equilibrium.

The balance line represents the position of the skier in a specific moment, which is not necessarily a balanced or stable position. Stability is achieved by aligning the balance line along the trajectory of force from the snow – that being the GRF.

At any moment the ground reaction force can be simplified into a single trajectory, and it is this single trajectory push from the snow that defines a balanced state when our COM is positioned along it. In essence, we are balancing against the GRF.

Ordinarily, we associate balance solely with gravity, or the line of gravity. However, this is not the case in skiing, where the push from the snow (ground reaction force) should not be confused with the line of gravity. They are not the same and are rarely ever aligned. In skiing, our balance reference is the GRF and not gravity.

The above paragraph is not intuitive but is of critical importance. Please take the time to think on it and understand it.

Figures 7 and 8 on page 12 depict this concept.

Takeaways: GRF: A Balance Perspective.

1. The GRF can be simplified into a single trajectory.
2. The GRF trajectory, represents the balance and stability reference.
3. The skiers balance line represents the position of the skier.
4. A skier aligns their balance line with the GRF to achieve a position of stability.
5. In essence, the ground reaction force (GRF) is what we balance against.

CHAPTER 4: UNDERSTANDING THE COMPONENTS OF GRF

4.1 Force Resolution and Composition

From the discussion of vectors, we know forces can be combined. In the same way, we can resolve a single force into multiple forces. GRF is a combination of numerous forces. Let's take a thorough look at the GRF by addressing each of its constituents and the effect they have on our skis, and in turn us.

At any given moment, GRF can be simplified to a single exact trajectory returned from the snow. This exact trajectory is determined by numerous forces including a perpendicular, centripetal, and most of the time, a slowing component. The perpendicular component opposes gravity and keeps us on the surface of the planet, or snow in our case, the centripetal component is responsible for turning us, and further resolution of the turning component exposes a slowing component which, well, slows us down.

Takeaways:

1. GRF is made up of numerous forces.
2. A component keeps us on the surface of the snow, a component turns us, and yet another slows us.

Let's examine each of the three components and how they work together.

4.2 Perpendicular: The Force That Keeps Us on the Surface of the Snow

We take for granted, and expect to stay on, the surface of the planet. However, a force is responsible for this, a reaction force that opposes our weight, commonly referred to as the normal force.

Recall back to the Figure 2, the normal force example where the table pushes back on the sandbag. Extrapolating to a skier on snow on perfectly flat terrain for example. The skier applies a force to the snow (their weight) and the snow applies an equal but opposing force to their skis. If no other forces are introduced, they remain upright and stationary. The normal force is the perpendicular component of the GRF that keeps the skier on the surface of the snow.

Generally, the snow surface is compacted to the point that it holds us on the surface, this is certainly true for all prepared surfaces. However, at times, specific conditions, fresh snow for example, allow us to sink into the snow to some degree. We sink until the snow is compacted to the point that it pushes back equal to the force we apply to it.

Let's add some momentum to our example by placing the same skier on a slope and on a straight run. Due to virtually no friction between the snow and the running surface (base) of the ski, the reaction force from the snow pushes perpendicular to the surface of the snow or the base of your skis, however you like to think about it (Figure 8). This last statement is an important detail, because to stay in balance, the trajectory of the snow's reaction force needs to pass through our feet and our centre of mass (COM). A practical example of this is when learners often fall backwards during their first attempt at a straight run or when getting off a lift for the first time.

Figure 7 GRF and Gravity are aligned.

Figure 8 GRF and Gravity are not aligned.

Takeaways:

1. A perpendicular component of GRF, called the normal force, resists our weight and keeps us on the surface of the snow.
2. We sink into soft snow until the snow is compacted to the point that it pushes back equal to the force we apply to it.

4.3 Turning Forces: Centripetal, Centrifugal

Continuing with our example above, the skier on a straight run will require a side force before any directional change is possible.

The component of GRF responsible for turning is classified as a centripetal force. To create this side force, the skis need to be placed at an angle to the direction of travel (across the direction of our momentum). To what degree we place the ski across the line of travel directly relates to the angle at which the GRF pushes on us. The angle we place the skis across our line of travel is referred to as a steering angle.

Centripetal force is a category of force that represents a centre-seeking force as objects travel in a circle.

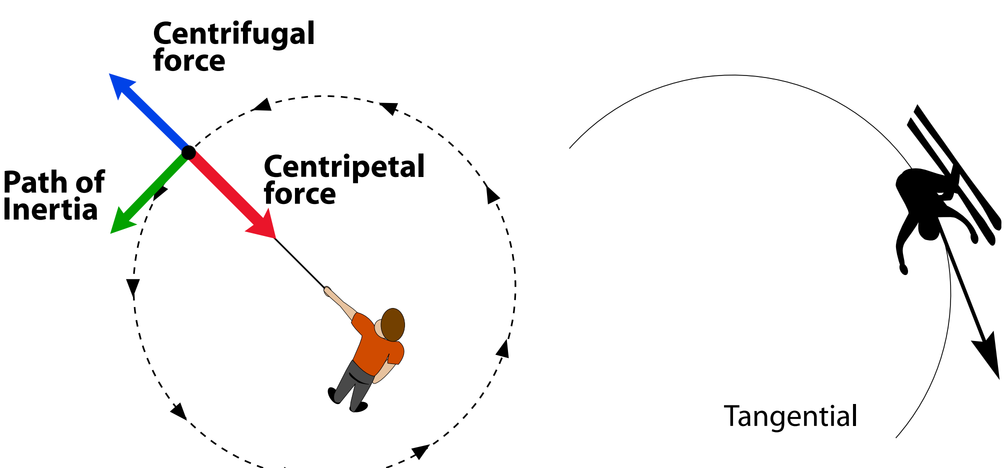
Examples of centripetal forces are all around us: the earth orbiting the sun (caused by gravitational attraction of the sun), a ball being swung around on a string (caused by tension in the string), a car going around a corner (friction on the tires) and a bobsled turning around a corner (caused by the banking of the track). The centripetal force from the snow, a reactionary force caused by the skier pushing on the snow as it pushes us towards the centre of the turn, is responsible for turning us.

Centrifugal force is a category of force that represents a centre fleeing, fictitious force, i.e., the force we feel and often referred to as the equal and opposite reaction to centripetal force. If we drive around a curve in our car or on a bicycle, or if we are engaged in a ski turn, we *feel* like we are being pushed to the outside of the turn. This apparent or *fictitious* force acting outward on a body moving around a centre is referred to as centrifugal force (centre fleeing). In skiing, it results from the inertia of our body, or the tendency to resist any change in its state of motion (recall Newton's first law).

The centrifugal force we feel is a reaction to the centripetal force. From our own perspective (or from our frame of reference), we feel centrifugal force, making it relevant and real. However, from an outside observer's perspective (or from an observer's frame of reference), there is no centrifugal force, there is only a centripetal force acting on the ski, making it turn, and therefore making the skier turn.

There is a counterintuitive aspect to address regarding the relationship between centripetal and centrifugal force. If the centripetal force is removed, the object will immediately travel at a tangent to the path of the circle around which it was travelling (path of inertia). It would *not* be ejected straight out from the centre as centrifugal force would lead us to believe or *feel*. The instant there is no centripetal force, centrifugal force disappears. At this point, Newton's laws again set us straight (pun intended). The direction of travel will continue in a straight line until a net force acts on the body.

Figure 9a and 9b. Centripetal, centrifugal, tangential



Physicists have derived a formula for centripetal force from Newton's second law, which we can examine in order to reveal some interesting facts:

Centripetal force = mass × velocity squared / radius

The only way to change centripetal force (turning force) is to change the speed or radius, they are the only two variables or manipulators effectively available.

The above equation implies that we would feel more forces when skiing by increasing the velocity (a squared relationship) than we would by reducing the radius of our turn (an inverse relationship). Go for a ski and play with speed and radius in your turn. Feel the effect that each contributes.

The above equation also implies we would feel more forces with more mass present. Test it out! Put something heavy in a backpack or wear a weighted vest and go for a ski. Experience the increased forces.

Without manipulating mass, as in the above drill, we only have two manipulators available to us to change the force of a turn – speed or radius!

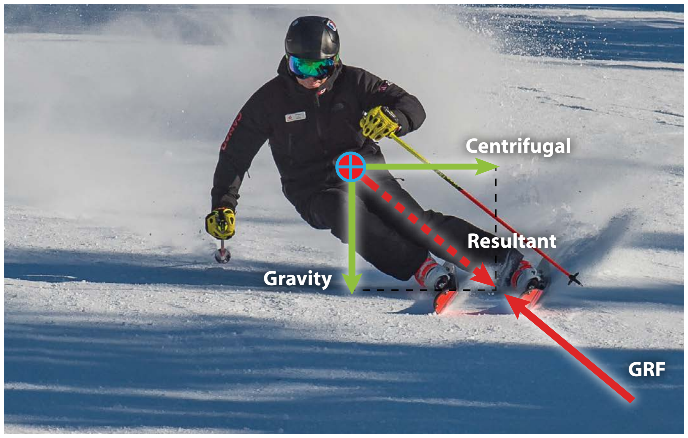
Additional information (3)

Takeaways:

1. The component of GRF that turns us is a centripetal force.
2. Centripetal force is a category of force that represents a centre-seeking force as objects travel in a circle.
3. In order to turn (create the centripetal force), we must place the ski across our line of travel. The steering angle can be small, but make no mistake, it is a mandatory requirement to turn.
4. Centripetal force (a centre-seeking force) is a real force, whereas centrifugal force (a centre-fleeing force and the force that we actually feel) is a fictitious force. Centrifugal force is equal and opposite to centripetal force.
5. If centripetal force is removed, the object will immediately travel at a tangent to the path of the circle around which it was travelling.
6. There are only two variables available to us to change the force of a turn – speed or radius!
7. The relationship between centripetal force and turn radius is inverse. The smaller the radius of the turn, the greater the centripetal force. If we reduce the turn size by half, the centripetal force doubles.
8. The largest factor in determining the magnitude of centripetal force is velocity, given the squared relationship between velocity and force as indicated above, i.e., if we ski twice as fast, the centripetal force quadruples.

Below, we graphically represent the forces in a ski turn, both from the perspective of what we feel, our internal reference frame, and through an external reference frame of what an outsider would observe.

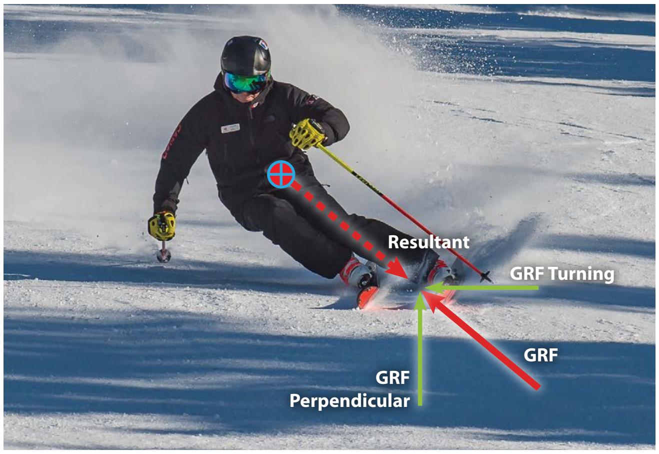
*Figure 10a – **INTERNAL** reference frame. What we feel in a turn.*



We feel a centrifugal force pulling us to the outside of the turn and we also feel gravity holding us on the surface of the snow. We also feel the resultant muscular tension predominantly in our outside leg and core keeping us upright and resisting the combined centrifugal and gravitational forces. Finally, we feel pressure under our outside foot, which is the snow pushing back on us (the GRF).

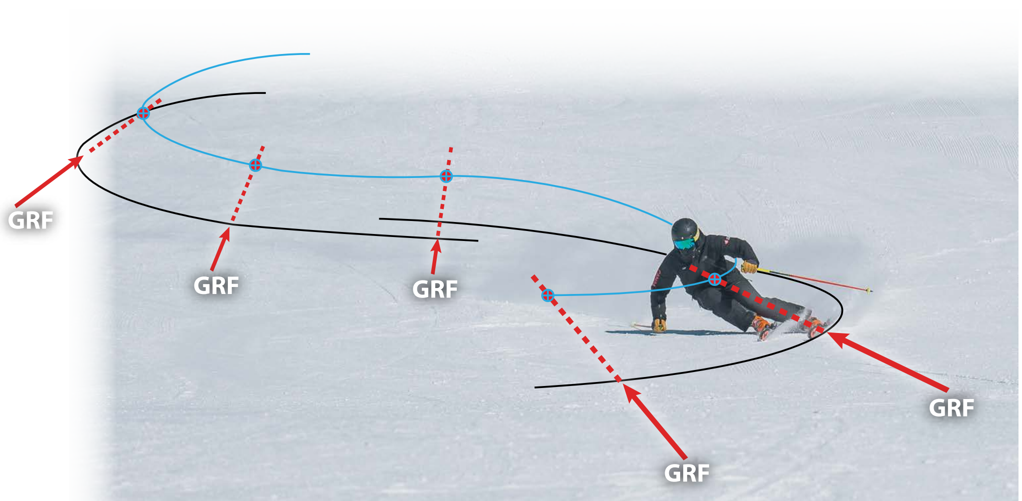
Gravity acting on the skier's mass combines with centrifugal force to form a resultant force R acting on the skier.
R represents the direction the skier is pushing on the snow.
GRF is the reaction force produced by the snow in response to R
The skier is balancing against the GRF

*Figure 10b – **OUTSIDE** reference frame. Representation of the actual forces.*



GRF-perpendicular – the snow's response to our weight (mass × gravity). This is the force that holds us on the surface of the snow.
GRF-turning – a centripetal force pushing our ski towards the centre of the turn. This is the force that is changing our direction, the one turning us. This centripetal force is what we feel as a centrifugal force in diagram 10a.
GRF – the exact trajectory of the snow's reaction to us pushing on the snow. It is the combination of the above two forces (perpendicular and turning).

Figure 11



Throughout a turn, GRF acts on COM at an angle to our line of travel, at each point in the arc, so the result is a curved trajectory.

4.4 The Slowing Component

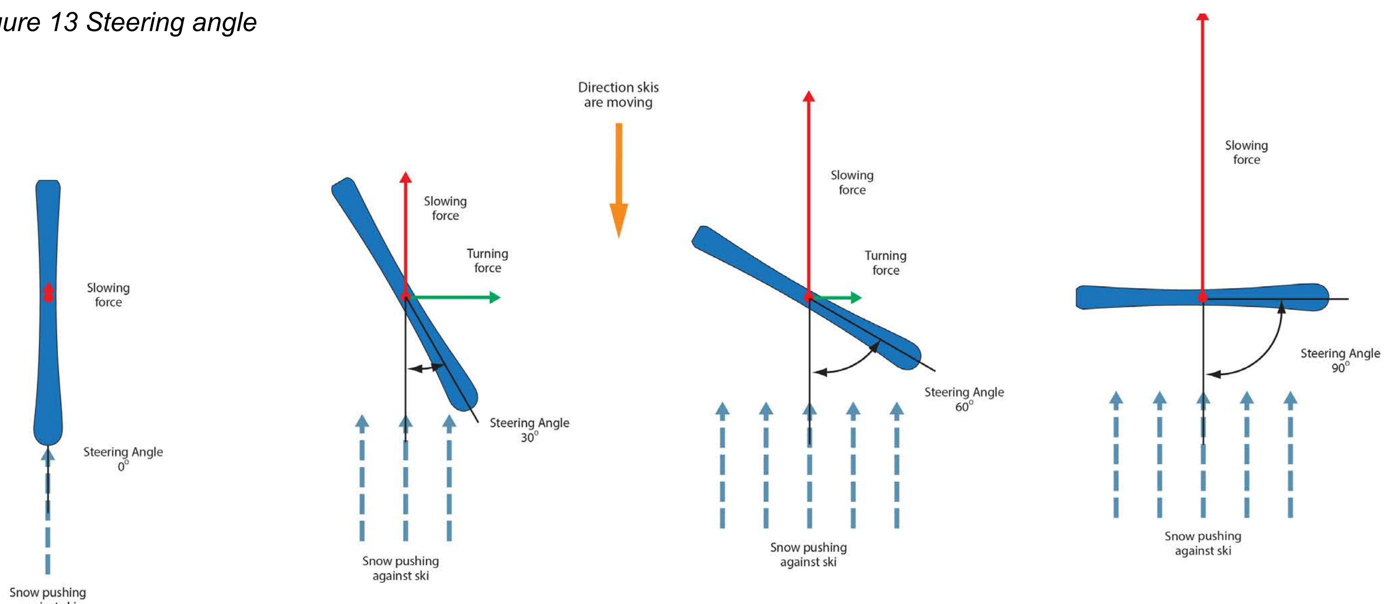
Let's continue with the example of our currently turning skier. We can examine or resolve the turning force even further to determine how much of the force is turning the skier and how much is potentially slowing them. We say potentially, as it's possible that there is no slowing component – a perfectly carving ski represents the situation with no slowing component.

GRF turning can be further resolved into two additional components: one that slows the skier, GRF-slowing, and one that turns the skier, GRF-lateral. The magnitude and direction of these components combined, serve to impact the skier's motion.

Figure 12

In addition, the examination and discussion of slowing and turning force components helps to clarify how we choose to define turn types. As can be seen in Figure 13 below, when we increase the steering angle of the ski, the slowing component also increases. The greater the slowing component, the more the ski is drifting. As the slowing component is reduced, the drifting of the ski is reduced. When the slowing component is at zero, the ski is carving.

Figure 13 Steering angle



Takeaways:

1. The turning component of the GRF can be further resolved into two additional components: one that slows the skier, and one that turns the skier.
2. When we increase the steering angle of the ski, the slowing component also increases.
3. The larger the slowing component, the more drifted the turn.

4.5 GRF - The Balance Reference

We established above (page 10, GRF: A balance perspective) that stability is achieved by aligning the skiers balance line along the trajectory of force from the snow – that being the GRF. **The trajectory of the GRF is the balance and stability reference.**

However, we need to consider how the snow pushes on us both front to back and side to side to understand the exact trajectory of the GRF. One viewed from the side (sagittal plane) and another viewed from the front (frontal plane). The GRF can be simplified into a single exact trajectory at any specific moment, but we need both reference planes to determine where that is.

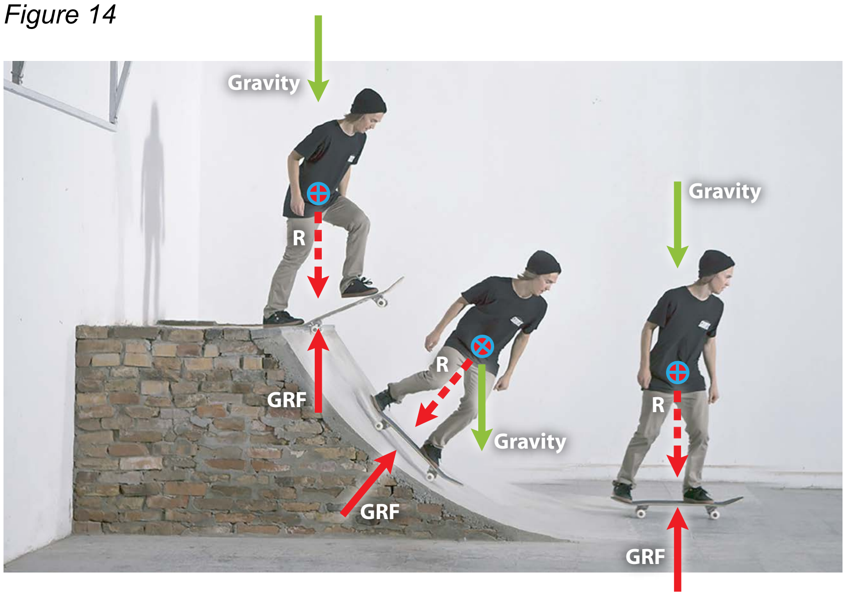
Takeaways:

1. The trajectory of the GRF is the balance and stability reference.
2. We need to consider how the snow pushes on us both front to back and side to side to understand the exact trajectory of the GRF.

4.6 Fore/Aft Balance

We touched on fore/aft balance in section 4.2, stating that due to the very slippery interface between the ski and the snow along the running length of the ski, the force from the snow always pushes up perpendicular to the running surface of the ski (ski base). Meaning that optimal balance sits perpendicular to the base of the ski when viewed in the sagittal plane – fore/aft balance. Hence, maintaining our base of support (feet) and centre of mass on this line represents the most effective position to control the ski. The below image of the skateboarder drop in illustrates this concept. In the second frame of the image the skateboarder is on an incline, and the ground reaction force acts perpendicular to the skateboard deck. The Skateboarder must move his COM perpendicular to the board deck to maintain balance. It is easy to visualize what would happen if the skateboarder in this sequence didn't adjust his balance line to match the inclination of the slope – he would very quickly fall backwards.

Figure 14



Due to the stiff nature and design of ski boots, and the fact that we attach them to a big lever, that being the ski, we have considerable margin to stray from this balance reference without falling. Additionally, we do intentionally at times pressure the front and tail of the ski to obtain certain results. Meaning that we will intentionally position our balance line fore or aft of the GRF trajectory as viewed in the sagittal plane. However, the GRF trajectory serves as the reference for balance, and we should endeavour to spend most of our time on and working from this reference. Our fore/aft balance is determined by how effectively we stay on and work off the trajectory of the GRF.

It’s important to realize that this reference line, perpendicular to the ski base, is continually changing. We, as skiers and teachers, need to understand and anticipate its trajectory – balance depends on it! We must anticipate the ever-changing pitch of the slope as we turn upon it – the pitch becomes steeper as we turn into the fall line and less steep as we come out of it. Additionally, think how changing terrain continually affects the angle at which the running surface of the ski interacts with the snow.

Takeaways:

1. As viewed from the sagittal plane, the GRF trajectory represents the fore/aft balance reference.
2. As viewed from the sagittal plane, the GRF trajectory sits perpendicular to the base of the ski.
3. As viewed from the sagittal plane, the GRF trajectory is continually changing due to the skis turning on the ski slope and terrain changes.
4. As viewed from the sagittal plane, we are in balance when the GRF trajectory passes through our feet and our centre of mass (COM). Or stated simply, a skier’s balance line and the GRF are aligned.

4.7 Lateral Balance

Our side-to-side balance is the more difficult one to get right. The lateral balance reference is again represented by the GRF trajectory, but now viewed from the frontal plane. Lateral stability is achieved when the skiers balance line, is aligned with the GRF as viewed from the frontal plane.

Laterally the skiers balance line is determined, as previously shown in Figure 10a, from the combination of gravity and centrifugal force, which we refer to as the Resultant.

As is true with the fore/aft balance reference, the lateral balance reference is continuously changing throughout the turn. As such, a skier must continuously adjust their balance line to maintain stability against the continuously changing balance reference of the GRF trajectory.

As we watch great skiers, we will notice that the degree of inclination at the bottom of the turn is much greater than at the top of the turn. This variance is due to the forces experienced at the bottom of the turn being much greater than those experienced at the top of the turn. The greater the force, the more we need to incline.

The reason for this is, the top part of turn has a component of gravity that diminishes the GRF pushing on us, and the bottom part of the turn has a component of gravity that increases the GRF pushing on us. We feel less force at the top and more as the turn progresses.

Additional information (4)

Additionally, skiers are often moving slower during the top portion of the turn than they are through the bottom portion due to speed increasing as we move through the fall line. As previously established, velocity has a material bearing on turning forces (page 14, takeaway bullet 8). The faster we travel through a turn, the greater the magnitude of the force.

Skilled skiers have a better ability to manage larger forces and increasing degrees of inclination. They can do this because of their athletic ability, their dedication to training, and ultimately, in the perfecting of their skiing skills.

Figure 15

Didier Cuche showing the extreme inclination (approximately 70 degrees!) required in order to manage the forces at high speed and remain in balance. His lean is required to be so great due to his velocity and radius – he's moving really fast. To be in balance, he must be this inclined, in order to oppose the trajectory of the reaction force from the snow - the GRF.

Takeaways:

1. As viewed from the frontal plane, the GRF trajectory represents the lateral balance reference.
2. Manipulating the Balance line inside or outside this reference is how we control balance and stability.
3. As viewed from the frontal plane (lateral balance), when the skiers balance line and GRF trajectory are aligned, the skier is in a stable and balanced position.
4. The greater the GRF, the more we need to incline.
5. We experience more ground reaction force at the bottom of the turn than the top of the turn.

4.8 Inclination - A Balance Analogy

The human balance system does not require the physicist's force equations to know what is going on. It simply feels these forces and is quite adept at manipulating the body to remain in balance.

As a skier *feels* they are being pulled towards the outside of the turn – a centrifugal force – they respond instinctively by inclining. The greater the force, the more we feel we need to incline against it.

In the context of a ski turn, we can think of this situation as a force balance equation – a tug of war. We have two teams, gravity and centrifugal force, and they are in a tug of war. Gravity pulls us to the inside of the turn and centrifugal force pulls us to the outside. Whatever degree our inclination, we use gravity as a counterweight on the inside of the turn to balance the centrifugal force pulling us to the outside of the turn.

As explained above, GRF increases in magnitude as the turn progresses, less at the top of the turn and greater at the bottom. As GRF increases, and we feel more centrifugal force, the line of inclination must increase to maintain the balance equilibrium between our counterweight of gravity and centrifugal force.

What happens if we do not get the gravity centrifugal equilibrium quite right? If we err on the side of gravity (Figure 16, G+) and incline too much, our balance line will be inside the trajectory of the GRF, and we will begin to fall to the inside.

We can either continue to allow ourselves to fall inside the turn or correct the situation with a recovery maneuver. When we use the inside ski, drag the inside pole, angulate, or reduce the turn radius to recover our balance, what we are doing is adjusting our current state of imbalance towards a state of balance – we are moving the balance line closer towards the trajectory of the GRF. Relative to the tug of war analogy, gravity had the advantage and we needed to quickly give the centrifugal team a helping hand (literally, if it was the inside hand).

If we err on the side of centrifugal force (Figure 16, C+) and maintain that bias, the skier will have effectively begun a movement toward the new turn. At what rate this happens depends on the degree of the imbalance.

4.9 Intentional Imbalance

Just as when walking – intentionally falling forward and catching ourselves with an outstretched foot – expert skiers intentionally unbalance and fall to the inside. They are adept at allowing their COM to fall just slightly ahead of the ski path that continuously catches them.

Said another way, we intentionally bias the side of gravity in the gravity/centrifugal force analogy, until such time as we choose to bring both the gravitational and centrifugal forces back into balance. The balanced state lasts only for a very short time before we intentionally unbalance the components again to trigger a *fall* (topple) back across our skis into the next turn and the cycle repeats.

An act of intentional imbalance is the mechanism by which advanced skiers allow the COM to fall farther inside a turn, as the means to propel the COM across the BOS and into a new turn.

Figure 16

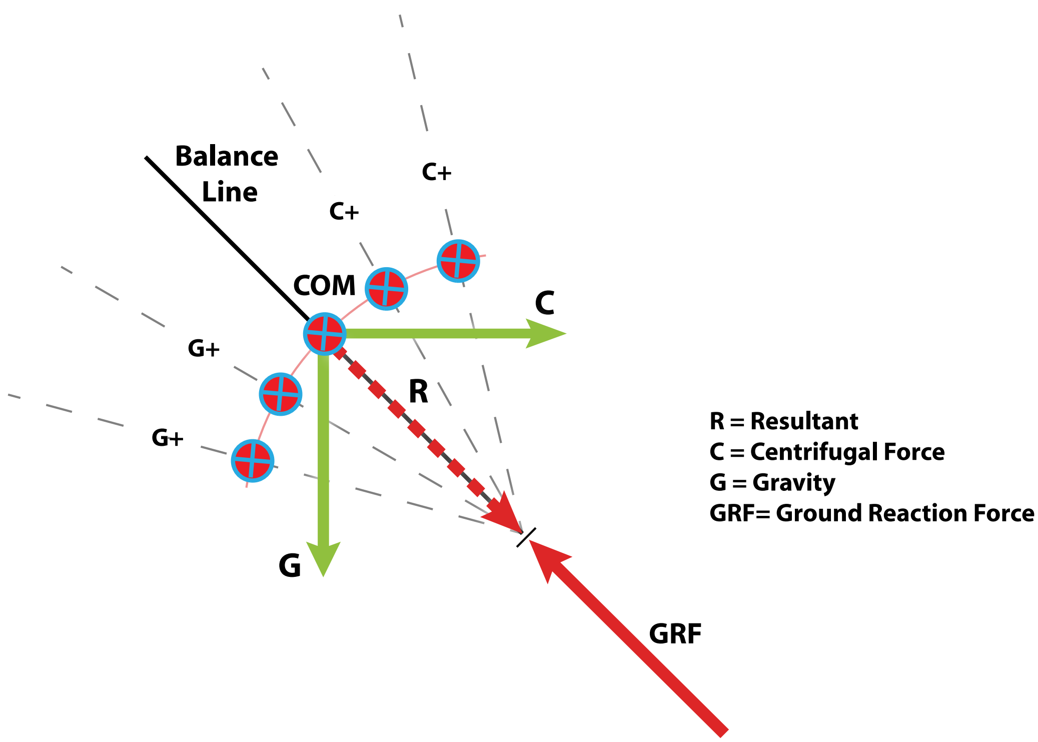


Figure 17 Ted Ligety extreme lean

Try it! Take a few runs and play with the degree of inclination.

Remember how the forces determining the amount of inclination are governed: by gravity, which we obviously cannot affect, and centripetal force, which we feel as centrifugal force. Centripetal force is governed by velocity and radius.

As you'll recall from above, skiing twice as fast quadruples the centripetal force. To establish a base reference, take a comfortable GS run. Then double the speed, but keep the radius the same, and observe the amount of inclination needed to stay in balance. Also observe where in the turn you require the most inclination, at the top or bottom part of the turn.

Additional information (5)

Takeaways:

1. Expert skiers constantly and intentionally imbalance themselves laterally through misalignment of the skier's balance line and GRF trajectory.
2. An act of intentional imbalance is the mechanism by which advanced skiers allow the COM to fall farther inside a turn, and as the means to propel the COM across the BOS and into a new turn.

Here we conclude the Physics of Skiing. The Skills Framework is the next document in the series.

Supplementary technical details and explanations on certain aspects are covered in the additional information section that follows, for those who are so inclined to dig into even more technical aspects.

Summary of Takeaways

Takeaways: Forces

1. Skiing is motion
2. We cannot have motion without force.
3. A **force** is a push or pull upon an object resulting from the object's interaction with another object (ski and snow).
4. The force of gravity pulls us down the hill.
5. Other forces facilitate speed and direction control.

Takeaways: Vectors

1. Forces have two key properties, magnitude and direction.
2. In physics, acceleration has a speed component, but also a directional component.
3. An object is accelerating if it is changing direction, even if remains at a constant speed.
4. If a body is changing direction, it is accelerating.
5. Forces are additive.

Takeaways: Internal and External forces

1. Forces in skiing can be divided into two categories: internal and external.
2. Internal forces are generated by the skier using their muscles. We can utilize these forces to influence our movements.
3. External forces are acting on the skier from outside the body. These forces are responsible for moving the skier.

Takeaways: Motion (Newtonian Mechanics)

1. The laws of motion govern the laws of skiing. Newton's laws define the rules of motion.
2. Newton's first law helps us define inertia, a force which keeps stationary objects at rest and moving objects in motion at constant velocity.
3. Newton's second law tells us that how much something moves is proportional to the size of the force acting on it. By manipulating our skis on the snow, we can manage the forces acting on us and therefore control our motion in order to achieve our objectives.
4. When skiers use their muscles or equipment to push on the snow, Newton's third law tells us the snow pushes back. So, yes, the snow really does push on us!

Takeaways: Forces and Motion; Gravity, Friction

1. The centre of mass is a specific point in the body where we consider the mass of the skier and equipment to be concentrated
2. Gravity is the primary force that provides skiers with momentum.
3. Gravity acts as two components: a component that pulls on us in a direction perpendicular to the surface of the snow and a component that pulls the COM down the hill.
4. Friction slows us down. We experience two types of friction: the friction between the ski and the snow and the air friction that pushes against a skier's body.

Takeaways: Momentum.

1. Momentum defines how much motion an object has (the quantity of motion).
2. Two variables determine an object's momentum, i.e., mass and velocity.
3. The momentum of an object will only increase or decrease when a net force is acting upon it.
4. On skis, we manipulate our momentum through the interaction of our skis with the snow, and therefore control how the snow pushes on us!

Takeaways: Ground reaction force (GRF)

1. Yes, the snow really does push on us.
2. The ground reaction force (GRF) is the snow's reaction to the force we apply to the snow.
3. The GRF can be simplified to represent the exact trajectory of the snow's push on us.
4. The GRF is the force that provides direction and speed control. It is the force that makes turning possible.
5. Through manipulating the GRF we get the snow to push on us in just the right way to achieve our desired outcome.
6. We control GRF through the interaction of our skis with the snow.
7. Gravity is the primary force that moves us. Motion yields momentum and we manage our momentum (speed and direction) through manipulation of GRF.

Takeaways: GRF: A Balance Perspective

1. The GRF can be simplified into a single trajectory.
2. The GRF trajectory, represents the balance and stability reference.
3. The skiers balance line represents the position of the skier.
4. A skier aligns their Balance line with the GRF to achieve a position of stability.
5. In essence, the ground reaction force (GRF) is what we balance against.

Takeaways: Understanding the components of the GRF.

1. GRF is made up of numerous forces.
2. A component keeps us on the surface of the snow, a component turns us, and yet another slows us.

Takeaways: GRF Perpendicular.

1. A perpendicular component of GRF, called the normal force, resists our weight and keeps us on the surface of the snow.
2. We sink into soft snow until the snow is compacted to the point that it pushes back equal to the force we apply to it.

Takeaways: GRF Turning. Centripetal, Centrifugal and Tangential

1. The component of GRF that turns us is a centripetal force.
2. Centripetal force is a category of force that represents a centre-seeking force as objects travel in a circle.
3. In order to turn (create the centripetal force), we must place the ski across our line of travel. The steering angle can be small, but make no mistake, it is a mandatory requirement to turn.
4. Centripetal force (a centre-seeking force) is a real force, whereas centrifugal force (a centre-fleeing force and the force that we actually feel) is a fictitious force. Centrifugal force is equal and opposite to centripetal force.
5. If centripetal force is removed, the object will immediately travel at a tangent to the path of the circle around which it was travelling.
6. There are only two variables available to us to change the force of a turn – speed or radius!
7. The relationship between centripetal force and turn radius is inverse. The smaller the radius of the turn, the greater the centripetal force. If we reduce the turn size by half, the centripetal force doubles.
8. The largest factor in determining the magnitude of centripetal force is velocity, given the squared relationship between velocity and force as indicated above, i.e., if we ski twice as fast, the centripetal force quadruples.

Takeaways: GRF Slowing

1. The turning component of the GRF can be further resolved into two additional components: one that slows the skier, and one that turns the skier.
2. When we increase the steering angle of the ski, the slowing component also increases.
3. The larger the slowing component, the more drifted the turn.

Takeaways: GRF - The balance reference.

1. The trajectory of the GRF is the balance and stability reference.
2. We need to consider how the snow pushes on us both front to back and side to side to understand the exact trajectory of the GRF.

Takeaways: Fore/aft balance.

1. As viewed from the sagittal plane, the GRF trajectory represents the fore/aft balance reference.
2. As viewed from the sagittal plane, the GRF trajectory sits perpendicular to the base of the ski.
3. As viewed from the sagittal plane, GRF trajectory is continually changing due to the skis turning on the ski slope and terrain changes.
4. As viewed from the sagittal plane, we are in balance when the GRF trajectory passes through our feet and our centre of mass (COM). Or stated simply, a skier's balance line and the GRF are aligned.

Takeaways: Lateral balance.

1. As viewed from the frontal plane, the GRF trajectory represents the lateral balance reference.
2. Manipulating the Balance line inside or outside this reference is how we control balance and stability.
3. As viewed from the frontal plane (lateral balance), when the skiers balance line and GRF trajectory are aligned, the skier is in a stable and balanced position.
4. The greater the GRF, the more we need to incline.
5. We experience more ground reaction force at the bottom of the turn than the top of the turn.

Takeaways: Intentional imbalance.

1. Expert skiers constantly and intentionally imbalance themselves laterally through misalignment of the skier's balance line and GRF trajectory.
2. An act of intentional imbalance is the mechanism by which advanced skiers allow the COM to fall farther inside a turn, and as the means to propel the COM across the BOS and into a new turn.

Additional Information

1. Newtons 2^{nd} law   F=ma

Mass is specific to the skier and their equipment, and so is different for everyone. It is fixed and we cannot change it. Acceleration, however, is a vector. We can manipulate and so it is therefore an important aspect of skiing. Being a vector, acceleration has two components, speed and direction. Therefore, we can have acceleration when traveling at a constant speed (with only direction change present) or when traveling in a constant direction (with only a speed change present).

Everyone on the earth has a gravitational acceleration acting upon them (the symbol in physics is ‘g’ – a constant on earth = 9.8 m/s^2) which, when multiplied by a person’s mass, defines the force of gravity acting upon that person (referred to as ‘weight’). So, from F = ma above, we can also state Weight = W = FG = mg.

EXAMPLE: A person weighing 73kg (160lbs) will have a weight on earth = 73 kg x 9.8 m/s^2 = 715 kg m/s^2 or Newtons (N), which is the unit of measurement commonly used to express force or weight under the SI system (International System of Units). On the moon, this same person would weigh approximately 1/6th this amount, as ‘g’ is only specific to the earth as mentioned above.

2. Momentum

It is the product of mass (kilograms) and velocity (meters per second). Thus, momentum is measured in kilogram meters per second (kg m/s).

Momentum, like velocity, is a vector quantity, having both magnitude and direction. A force applied to a body can change the magnitude of the momentum or its direction or both.

In practical terms, the momentum of an object will only increase or decrease when a net force is acting upon it, because the force will cause it to accelerate or decelerate and to have an increase or decrease in velocity.

Expressed mathematically, p = mv, where ‘p’ = momentum, ‘m’ = mass and ‘v’ = velocity.

EXAMPLE: A person weighing 73kg (160lbs) and traveling at a speed of 10m/s (22.4mph) will have a momentum = p = 73 kg x 10 m/s = 730 kg m/s.

We can prove that If we change our momentum this will result in a change in force over time.

Using our formula above for momentum $\Delta p = m\,\Delta v$ (where Δ is the symbol mathematicians use to represent “a change in”). Let’s call this equation #1. We also know from Newton’s 2nd law that F= ma where m= mass and a = acceleration. Finally, we can express acceleration as a change in velocity over time, or $a = \Delta v/t$.

Substituting, $F = ma = m\Delta v/t$. Re-arranging, $\Delta v = Ft/m$. Finally substituting this back into equation #1 we obtain: $\Delta p = m\,\Delta v = m \times Ft/m$. With mass cancelling $\Delta p = Ft$, or in words, a change in momentum results in a change in force over time.

3. Centripetal force

Recall Newton’s Second Law, F = ma where ‘F’ = force, ‘m’ = mass and finally, ‘a’ = acceleration.

However, in this case,

Fc = mac, where subscript ‘c’ refers to centripetal

As we know, velocity is a vector specifying how fast a distance is covered and the direction of the movement. Since the velocity vector (the direction) of a body changes when it moves in a circle, there is an acceleration. This acceleration is referred to as centripetal acceleration and can be expressed

as $ac = v^2/r$ where ‘a ’ refers to centripetal acceleration, ‘v’ refers to velocity and ‘r’ refers to radius of the circle.

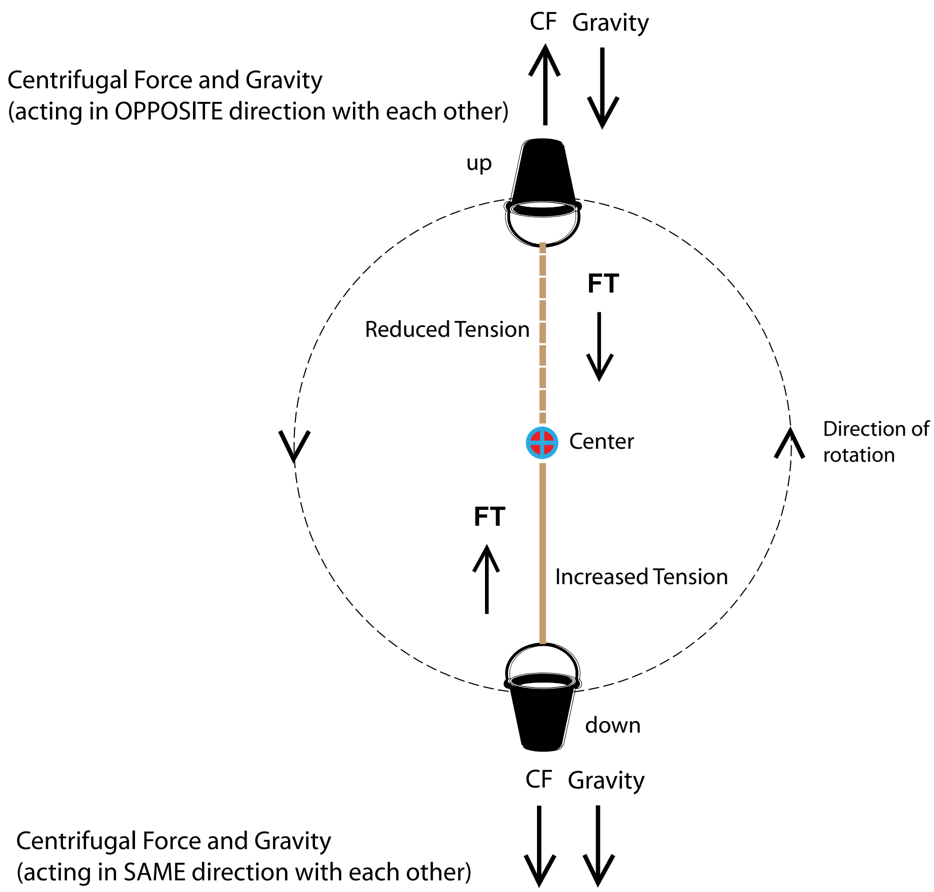
Therefore, $Fc = mac = m(v^2/r) = mv^2/r$,

So, Centripetal force = mass × velocity squared / radius ($Fc = m v^2/r$)

4. Inclination forces

This concept can be understood more easily by visualizing the motion of a bucket of water being swung around on a string in the vertical plane. The tension in the string (in skiing this is our GRF) is greater at the bottom (6 o'clock) position than at the top (12 o'clock) position.

Figure 18



In order to maintain the centripetal force required for the object to continue to travel in a circle at the top of the circle, the force of gravity is helping (force of gravity and centripetal force point in the same direction). Therefore, less tension in the string is required at the top.

In order to maintain the centripetal force required for the object to continue to travel in a circle at the bottom of the circle, the force of gravity is hindering (force of gravity and centripetal force point in opposite directions). Therefore, more tension in the string is required at the bottom.

In figure 18) the sum of the forces (FT=Force of Tension and FG=Force of Gravity) must always equal the centripetal force ($Fc=mv^2/r$).

At the TOP of the circle you can see FT points straight down and FG , as always, points straight down. As always, Centripetal force points toward the centre. If we adopt the convention that a force pointing towards the centre is positive, then $mv^2/r = FT+FG$. This equation can be re-arranged to calculate the Force of Tension at the top of the circle:

FT (top of circle) = $mv^2/r - FG = mv^2/r - mg = m(v^2/r - g)$

Alternatively, At the BOTTOM of the circle you can see FT points straight up and FG, as always, points straight down. As always, Centripetal force points toward the centre. Once again, if we adopt the convention that a force pointing towards the centre is positive, then mv^2/r = FT-FG. This equation can be re-arranged to calculate the Force of Tension at the bottom of the circle:

FB (bottom of circle) = mv^2/r – FG = $mv^2/r + mg = m(v^2/r + g)$

Let's relate this back to skiing! Obviously, there is no string to have tension in when we are turning. We are however, in contact with (and pushing against) the snow which is exerting a GRF onto us as explained above.

By way of example, let's say we have a 150lb (68kg) person travelling at a speed of 34mph (15m/s) turning with a 15m radius as in figure 18 above.

Per the above equations:

FT (top of circle) = $m(v^2/r – g) = 68(15^2/15 – 9.8)$= 354N, or expressed as G's = .53G's (recall G's = multiple of your weight)

FB (bottom of circle) = $m(v^2/r + g) = 68(15^2/15 + 9.8)$= 1,687N, or expressed as G's = 2.53G's! (Recall G's = multiple of your weight)

The above example is theoretical, however, as it is conducted in the vertical plane and obviously, we do not ski in a vertical plane (or at least not on purpose!). The force of gravity must be adjusted to account for slope of the hill. Adjusting Similar to the calculation used for inclination, if the slope of the hill is say, 25 degrees we would then substituting $g\sin\theta$ in place of g above, obtaining :

FT (top of circle) = 739N, or expressed as G's = 1.11G's, and,

FB (bottom of circle) = 1,302N, or expressed as G's = 1.95G's

G-Force:

The more we incline to balance against the reaction force produced by the snow, the greater the force we feel. We can use g-force to represent how much force we feel.

For illustrative purposes, we can express the force we apply to the snow (R) as multiple of the skier's weight. A force of one 'G' will be equal to 1 x the weight of the skier. A force of two 'Gs' will be equal to 2x the weight of the skier and so on.

Using mathematics, we can determine how many Gs are experienced by the skier at differing inclinations angles:

5. Inclination – Lean angle.

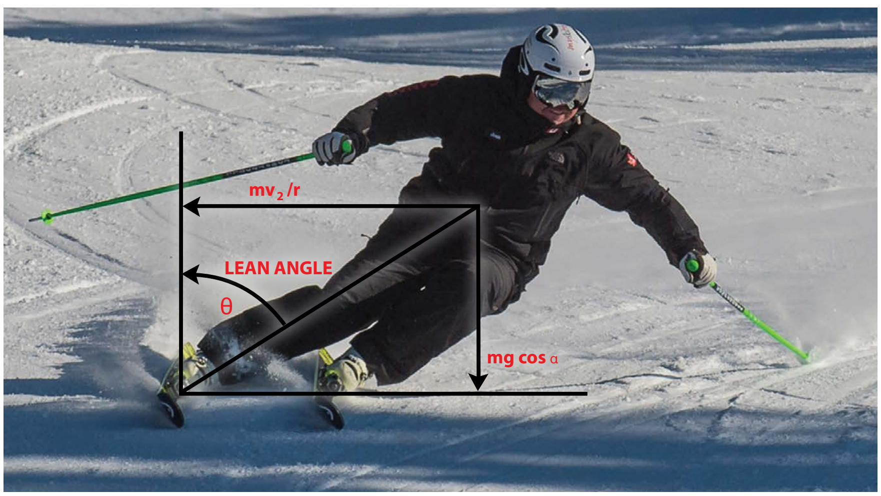
G = RcosΘ, where G = our weight (=force of gravity acting upon our mass), R = Resultant Force acting upon the skier and Θ is our inclination angle (=lean). Re-arranging, we obtain R = G/cosΘ. The table below represents the force (expressed in 'Gs' or multiples of the weight of the skier) at various degrees of inclination.

Inclination (Θ) (Degrees)	**Force** (Gs)	
0	1,0	
20	1,1	
30	1,2	
45	1,4	Expert skier
60	2,0	
70	2,9	Elite – WC Racer

How much do we lean in:

Using the equations for Newton's First Law and Centripetal Force as discussed previously, we may construct the following diagram below depicting lean angle and how it relates to the forces at play.

Figure 19 Lean Angle



Note: We feel Centrifugal force shown (from centre of turn outward) is equal and opposite to centripetal force = mv^2/r as showing in Figure 19.

From the diagram above, the lean angle, "Θ", can be stated as follows:

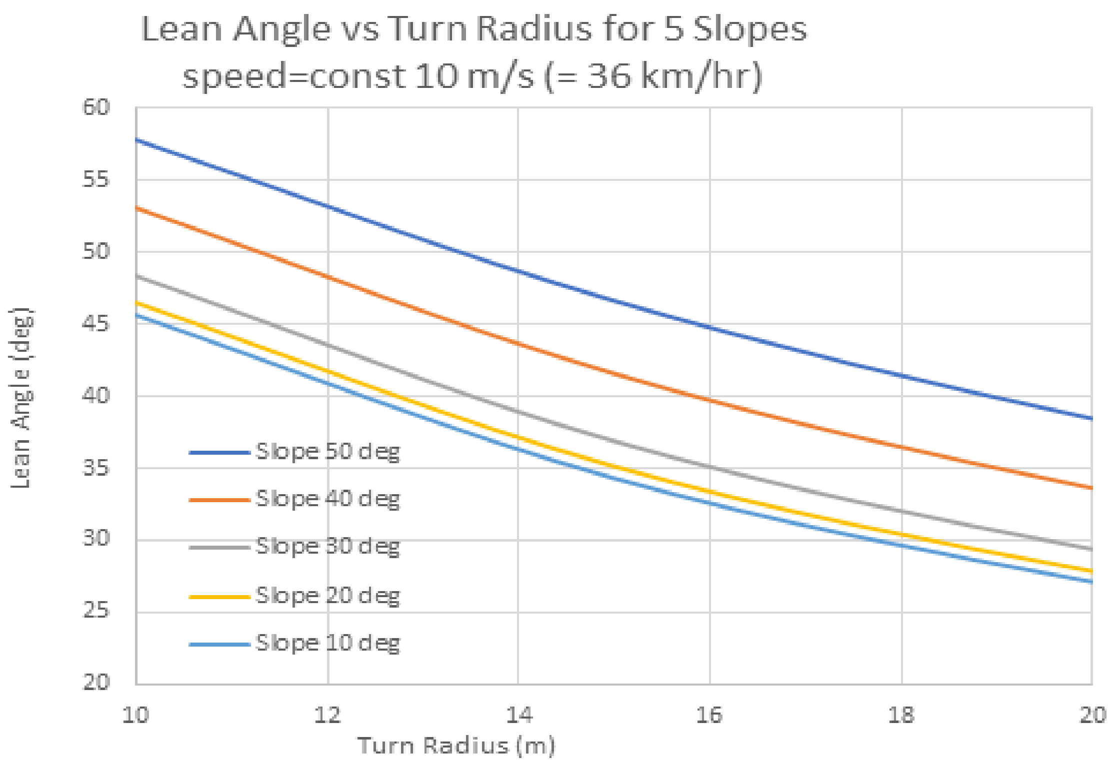
TanΘ = opposite / adjacent = $(mv^2/r) / (mg \cos\alpha)$, where "α" = slope of the hill.
With mass cancelling, TanΘ = $v^2/ rg \cos\alpha$
Solving for Lean Angle Θ,
Θ = tan-1 $(v^2/rg \cos\alpha)$

Using the above formula, we can plot graphs to present some possible lean angles for various turn radii at varying slopes (while holding speed constant) and at varying speeds (while holding slope constant).

For context when reviewing the graphs below: beginner hills generally range from 6 degrees to 25 degrees slope, intermediate 25 degrees to 40 degrees slope and expert 40+ degrees slope. In terms of speed, recreational skiers generally ski at speeds between 16 km/h (4.5 m/s) and 32 km/h (8.9 m/s), whereas a world cup GS racer will average around 80 km/h (22 m/s).

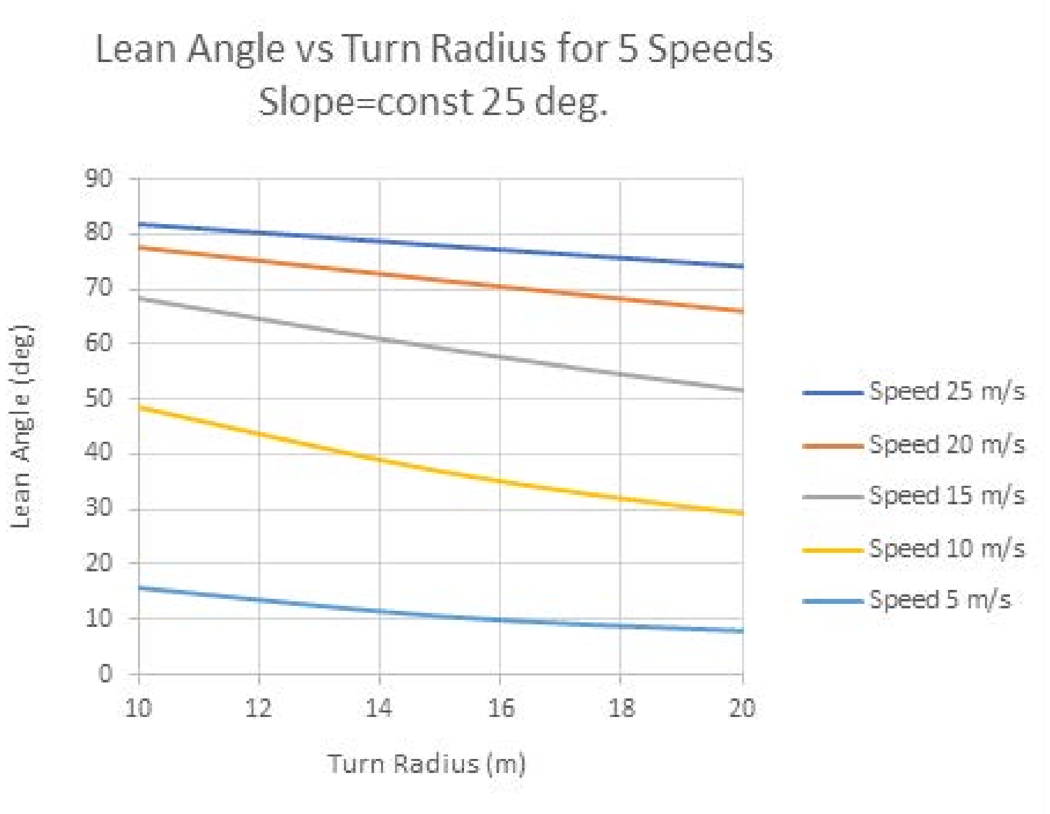
We can see from the graph below of Lean Angle vs Turn Radius at varying slopes (holding speed constant) that the lean angle dramatically increases as slope increases for any given turn radius. The lean angle also increases as the radius of the turn gets smaller (again holding speed constant). This shift is expected in light of the inverse relationship between centripetal force and turn radius outlined above.

Figure 20



We can also see from the below graph of Lean Angle vs Turn Radius at varying speeds (holding slope constant) that the lean angle increases more dramatically as the speed increases than it does as the radius of the turn decreases. This shift is expected in light of the squared relationship between centripetal force and speed outlined above.

Figure 21



Given the ‘squared’ relationship, the principal factor in determining how much we lean in is our speed. The faster our speed, the greater the lean angle must be to stay in balance.

The steeper the slope of the hill – the greater the lean angle must be to stay in balance.

The greater the speed – the greater the lean angle must be to stay in balance.

The bottom line: the amount we lean in is situational. Both experts and beginners alike will find themselves in a great variety of situations.

TRY IT! Take a few runs and be sensitive to the lean angle required to stay in balance. Try various slopes, turn radii and speeds and observe the required lean angle. Change only one of the three variables at a time in order to get the most out of the experiment.